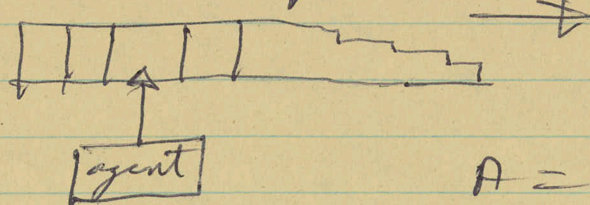
V-T-1

c 1937 Turing Machine (give a mathematical def of computation - algorithm
Idea: Turing Machine

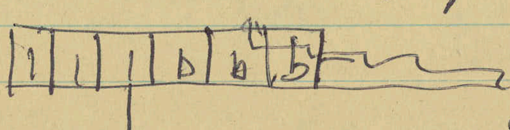
environment



→ infinite tape

A = alphabet (finite)

Possible agent actions
1) replace a symbol in n^{th} cell by another symbol
2) He can change his internal state of mind (He has a finite # of available internal states)
3) Make a move left, right, or stay put



TM (A, Q, P, S, M) quintuple

$Q = \{a_1 \cdots a_n\}$ set of states

A

$P(a_8$

Print $P: Q \times A \to A$, $P(a_3\, q_5) = a_7$

State $S: Q \times A \to Q$

move $M: Q \times A \to \{-1, 0, 1\}$

Recursively enum sets

Recursive functions
give this machine a number
Choose an alphabet $A = \{b, \cancel{x}, 1\}$

A configuration with $\#$ n 1's and then all b's.

Machine ~~states~~ starts in left most cell in state q_0

q^*
a^*

$$P(q^* a^*) = a^*$$
$$S(q^* a^*) = q^*$$
$$M(q^* a^*) = 0$$

Suppose eventually machine stops with number m

Def A function f $N \to N$ is called <u>partial recursive</u> if there is a

turing machine such that when started on the tape with $\# n$ on it — it halts with m on its tape $\longleftrightarrow$
$f(n) = m$

In number n in A will eventually find out If not you <u>never</u> will.

Def: A is recursive if both A an $N - A$
$= \bar{A}$ are rec

decideable

TM (universal Turing machine)

diagonal set $C = \{n \mid n \in W_n\}$

Claim C is r.e.

Claim $\bar{C}$ is not r.e. (hence C is not recursive)

Proof: Suppose $\bar{C}$ is r.e. Let $\bar{C}$ be W_m

$m \in \bar{C} \Leftrightarrow$ iff $m \in W_m \Leftrightarrow m \in C$

Godels Incompleteness theorem

Lemma For every r.e. set A there is a formula $\varphi(x)$ of arithmetic such that $n \in A \Longleftrightarrow \vdash_{\text{provable}} \varphi(\bar{n})$

Hilbert 10th

Diophantine

$x^2 + y^2 = 1$

$3xy^2 + 5x^3y - 13 = 0$

$\cancel{P(x, y)}$

$P(x_1, x_2 \ldots x_n) = 0$

Doesnt have a solution in the integers

Lemma for any r.e. set ~~A~~ we can find a Diophantine eq $P(x, y_1 \ldots y_m)$

$n \in A \iff P(n, y_1 \ldots y_m) = 0$

has a solution in the integers

~~That is no~~ There is no algorithm

for deciding whether a Diophantine equation ~~in the~~ has a solution in the integers or not

4/col John Landry

Poly Planes

OTIS/Switzer